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Advances and prospects in the accurate modeling of precession-nutation from VLBI solutions

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Introduction - 1

- ***The current conventions endorsed by the IAU and the International Union of Geodesy and Geophysics (IUGG) specify the Earth orientation in space by a set of 5 Earth orientation parameters (EOP), which include the traditional precession-nutation angles***
- **According to the IAU Resolutions in force, precession-nutation (PN) can be expressed either in the ecliptic paradigm, as the longitude ψ and obliquity ϵ of the ecliptic, or in the celestial intermediate pole (CIP) paradigm, as *celestial pole offsets (CPO)* – defined as the deviations *dX* and *dY* of the CIP coordinates with respect to its location computed from the conventional IAU2000 nutation and IAU2006 precession theories**
- **The CPO pair (or alternatively d ψ, d ϵ) can be accurately determined only from observations using the geodetic VLBI technique. Most of the CPO determinations come from the analysis of the 24 h long VLBI "R" sessions, performed twice a week *(R1 & R4)***
- **Most solutions provide the CPO pair in terms of dX and dY, and use the official IAU models IAU2000/2006 as *a priori* in the computations.**

Introduction - 2

- *Each main Analysis Center (AC) of the International VLBI Service for Geodesy and Astrometry (IVS) computes the EOP routinely on a session-wise basis (EOP-S), with a latency of a few weeks - higher for the combined solutions by the IVS Combination Centres*
- **CPO determinations are irregularly spaced and there are multiple results for days on which there were simultaneous sessions. A few ACs and the Earth Orientation Centre of the International Earth rotation and Reference systems Service (IERS) produce and release series of daily EOP, the CPO pair being time–densified by using sophisticated algorithms.**
- **The weighted root mean squared (WRMS) of the CPO time series is the main indicator of the variance uncovered by standard models and thus of their accuracy**
- ***Despite the increasing of accuracy of the VLBI technique in the last decades, the WRMS of each CPO series is hardly decreasing. An illustrative reference value can be set at 200 µas***
- **Therefore, the precession-nutation models are far from meeting the stringent, present accuracy requirements,** ***e.g. those set by the Global Geodetic Observing System of the International Association of Geodesy (GGOS/IAG) – 1 mm or 33 µas***

Introduction - 3

- **In this context, Resolution B2 of the IAU in 2021 and Resolution 5 of the IAG in 2019 encouraged the prompt improvement of the Earth rotation theories and models, regarding their accuracy and consistency**
- **The IAU/IAG Joint Working Group on Improving Theories and Models of the Earth's Rotation (JWG ITMER) is contributing to the implementation of the said resolutions. Matters related to improving the current IAG/IUGG - IAU PN models are prioritized**
- **A complete update of the theories in force would improve both accuracy and consistency. However, it seems unfeasible at the short-term, and thus the option emerges of improving the models by supplementing them with suitable corrections**
- **It appears to be feasible in the light of the many promising results obtained up to now *(e.g., among articles Petrov 2007, Koot et al 2008, Malkin 2013, 2016, Belda et al 2016, Gattano et al 2017, Belda et al 2017, Nurul Huda et al 2020, Zhu et al 2021, Ferrándiz et al 2022; and presentations at main meetings as Heinkelmann et al 2018, Ferrándiz et al 2019, 2020, 2021, etc.)* and was recommended by the 2019 IERS/GGOS Unified Analysis Workshop**

Purpose

- **In this presentation we assess to which extent we can reduce the unexplained CPO variance by applying corrections to the PN models**
- **Benefits of corrections to the different components of CPO models – precession, forced nutations, free nutations – are shown separately and jointly as well**
- **Results are mainly derived from well-known VLBI solutions obtained by single IERS ACs, and combined solutions as well.**
  We used only solutions with > 4,000 data points in the period 1984-2021 covering two full cycles of the lunar node, namely gsf2020a, usn2021c, opa2021a, bkg2020a, ivs20q2X, gsfc20q2, bkg20q2, dgfi20q2, ivs19q4X, downloaded from CDDIS or BKG web sites
- **That approach should minimize the risk of numerical artifacts due to the computations embedded in the obtaining of daily solutions –** ***IERSC04, usno-finals, and JPL2 are used for comparisons***

Corrections to the precession model
A sample of results and prospects of model improvement

(following Ferrándiz et al, IVS General Meeting 2022 proceedings)

- **IAU2006 precession model consists of the two components of the P03 theory (*Capitaine et al 2003*), namely the precession of the equator and that of the ecliptic**
- **Two new main features of it are:**
  - ***introducing 5-degree polynomials* to model the two former components,**
  - **Considering the Earth's *dynamical ellipticity H a linear function of time*, derived from the J_2 rate inferred from satellite laser ranging (SLR) observations since 1976**
- **Next we address some issues related to the two former features:**

1. ***Accuracy of the precession polynomials.*** **Their dominant components are the linear terms, we focus on updating rates and offsets**
2. ***Inconsistencies between IAU2006 & IAU2000 nutation model, where H is constant***
3. ***Other issues arising from the actual J_2 variation inferred from observations versus the simple model used in P03***

Correcting precession: Simple linear fit works quite well

We derived corrections to biases and rates of each CPO solution in the period 1984-2021 ***(but finals, which start in 1992, and JPL2, in 1998)***

All solutions referred to ITRF14 and ICRF3 ***– but JPL2 referred to JTRF***

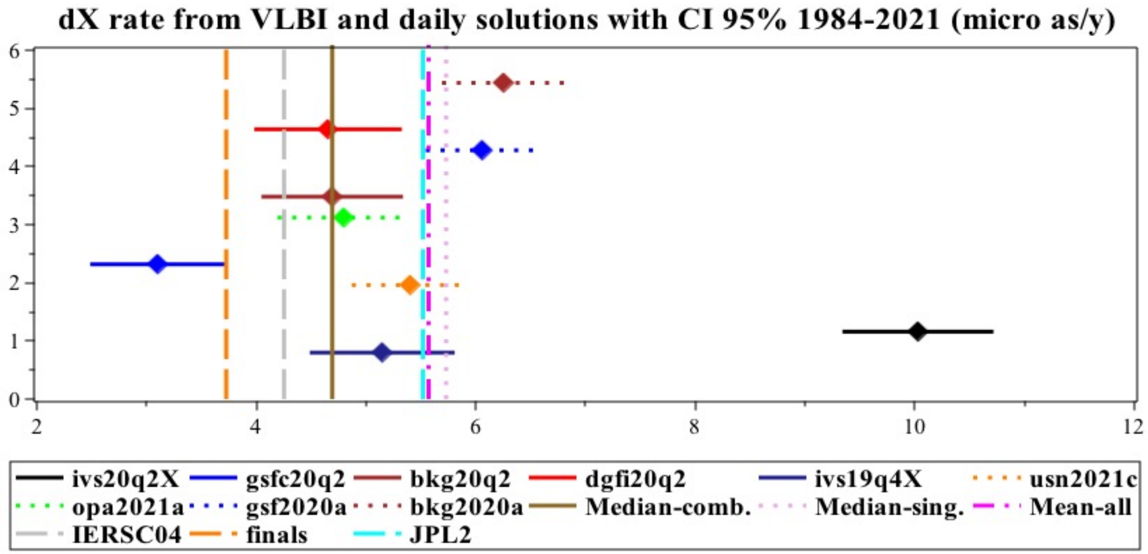
CPO	Solution	No. Obs.	Trend µas/y	Offset µas	σ - trend	σ – offset	WRMS raw data	WRMS detrend
dX	bkg2020a	5989	6,253	44,65	0,287	4,03	214,6	172,6
dX	gsf2020a	6431	6,058	22,38	0,262	3,66	196,0	166,1
dX	opa2021a	7027	4,797	-28,26	0,308	4,34	211,1	205,8
dX	usn2021c	5613	5,402	16,64	0,272	3,73	183,4	160,8
dX	bkg20q2	4209	4,691	2,18	0,329	4,10	177,8	167,3
dX	dgfi20q2	4034	4,653	-1,46	0,343	4,40	172,6	162,4
dX	gsfc20q2	4229	3,101	19,95	0,313	4,03	176,9	167,6
dX	ivs20q2X	4266	10,029	-27,55	0,351	4,89	205,0	168,0
dX	ivs19q4X	4215	5,147	16,37	0,337	4,38	173,1	152,9
dX	IERS14C04	13879	4.253	36.01	.166	1.76	180.2	164.8
dX	finals	10910	3.725	-28.73	.175	2.11	150.1	147.0
dX	JPL2	8729	5.518	22.18	.239	3.17	174.6	147.7

Results for dX

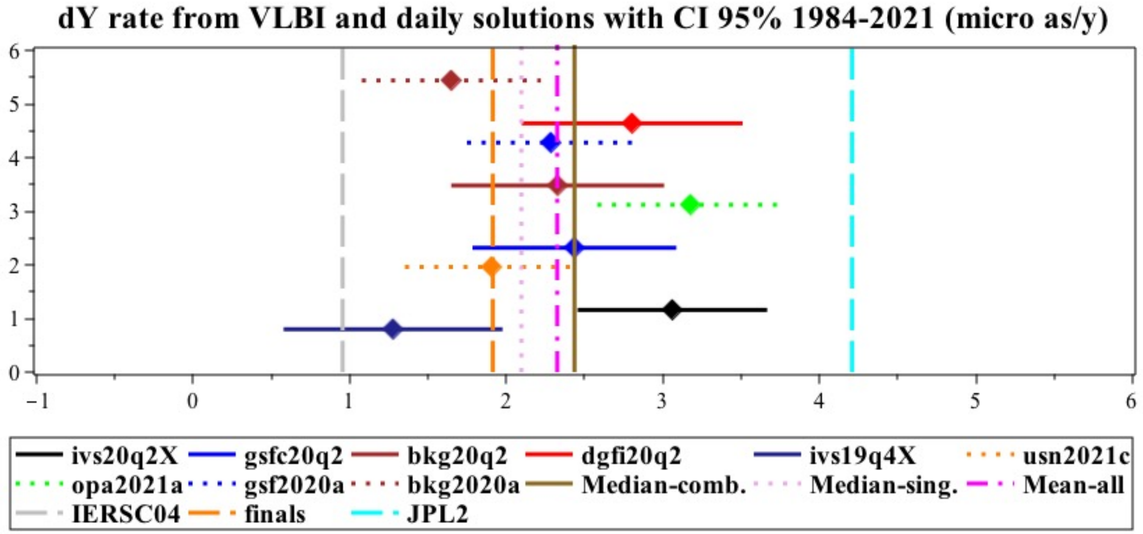
Highlighted the lowest WRMS after detrending, in each category (*single, combined, daily*)

Offsets in J2000.0; units µas, years

Correcting precession: mean dX variance reduction is about 20% (dY->12%)



dX	Mean - VLBI only	Median – VLBI only	Mean - all	Median - all	Min.	Max.
WRMS – raw data	190.1	183.4	184.6	179.0	172.6	214.6
WRMS detrended	169.3	167.3	165.3	165.5	152.9	205.8
% variance lowering	20	22	19	19	5	35
Rate (µas/y)	5.57	5.15	5.30	4.97	3.10	10.03
Bias (µas)	7.2	16.4	7.9	16.5	-28.3	44.65



dY	Mean - VLBI only	Median – VLBI only	Mean -all	Median - all	Min	Max
WRMS – raw data	183.2	183.4	179.2	176.4	172.6	214.6
WRMS detrended	171.3	172.8	166.4	167.0	152.9	205.8
% variance lowering	12	12	14	13	1	19
Rate (µas/y)	2.32	2.33	2.33	2.31	0.96	4.21
Bias (µas)	-83.6	-85.9	-86.5	-87.3	-129.6	-14.4

Precession:

Prospects of improvement by correcting the linear trends of the model

The refinement and update of the CPO trends and offsets values allows the reduction of WRMS of VLBI series in 10-15 *μas*

Potential actions to be taken:

1. ***Determine, agree and apply corrections to dX and dY trends***
   Using the medians/means of a sample of VLBI solutions or just the IVS one may be a first choice to start estimation and validation; initial values ranges may be

   dX trend correction -> about 5.524 to 5.628 *μas*/y
   dY trend correction -> about 2.255 to 2.381 *μas*/y

2. **Get more insight into the origin of the offsets larger variability – *it might be due to effects of differences in processing strategies difficult to identify and thus take longer***

Precession 2: results on inconsistencies

1. **Inconsistencies between IAU2006 and IAU2000**

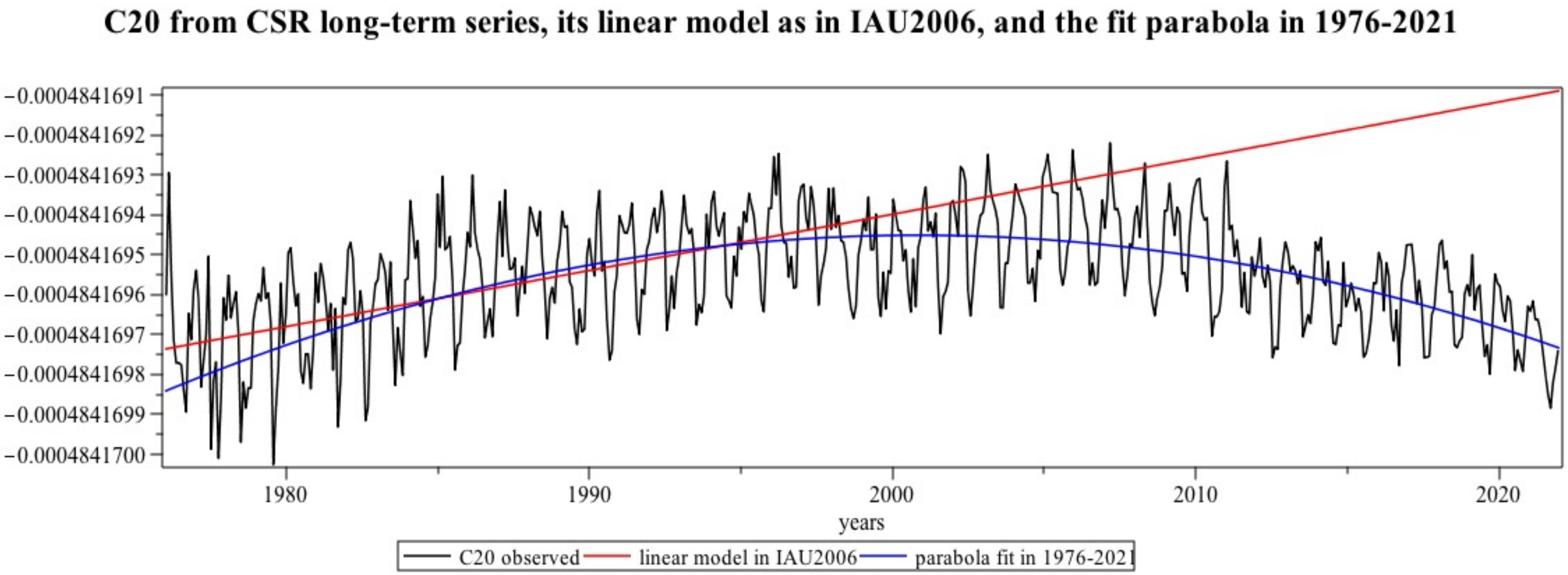
- **These inconsistencies arise from the fact that IAU2006 uses a constant J_2 rate unlike IAU2000, which considers the precession parameter as a constant; besides, IAU2006 and IAU2000 use different values for the obliquity and the "precession constant" *(i.e., rate of longitude)***
- **Inconsistencies may be overcome by adding a few correction terms to the nutation model *(Escapa et al 2017, Escapa & Capitaine 2018)***
- **Their magnitude is very small , but they contain terms growing linearly with time.** ***A few years ago it was agreed not to apply those corrections until the time of adopting others of higher magnitude***

$$
\begin{aligned}
dX =& (-6.2 + 15.4\,t)\sin\Omega - (0.6 + 0.6\,t^2)\cos\Omega \\
& + 1.4\,t\,\sin(2F - 2D + 2\Omega), \\
dY =& (0.8 - 25.4\,t)\cos\Omega - (0.8 + 0.3\,t^2)\sin\Omega \\
& - (0.3 + 1.8\,t)\cos(2F - 2D + 2\Omega).
\end{aligned}
$$

*Units μas and **cy***

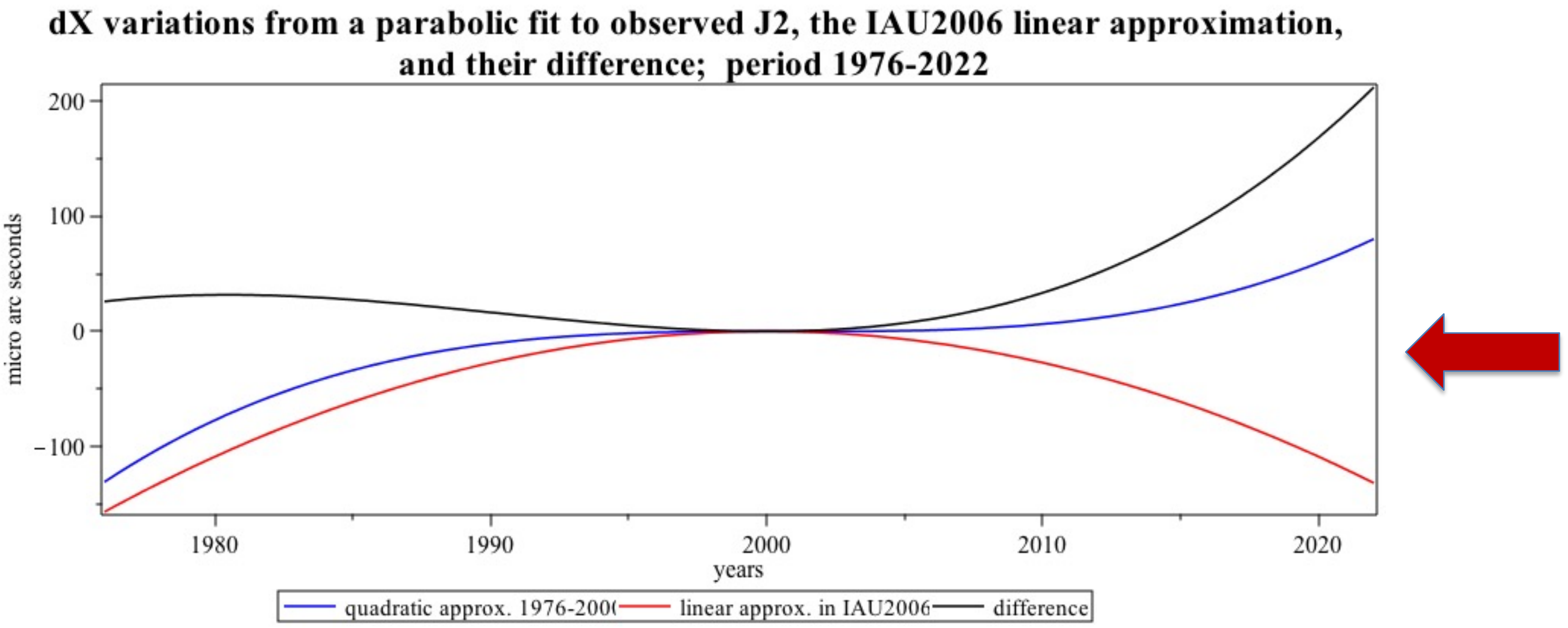
Precession 3: effects of the assumptions on the H variation

- **IAU2006 adopted *a J_2 trend of -3.001 E-9 /cy*, inferred from the observed Earth's oblateness change. That value is listed in the *System of Astronomical Constants 2009-2012, adopted by IAU 2012 Resolution B2*, and is equivalent to a trend $H_1 = H_0$ (-2.7719 E-8) /y , H_0 being the corresponding reference value of the dynamical ellipticity**
- **However, the trend started to change sign by 1997, and along the last 20 years it is reversed. At present, the observed $J_2 = -C_{20}$ is better represented by a second-degree polynomial**



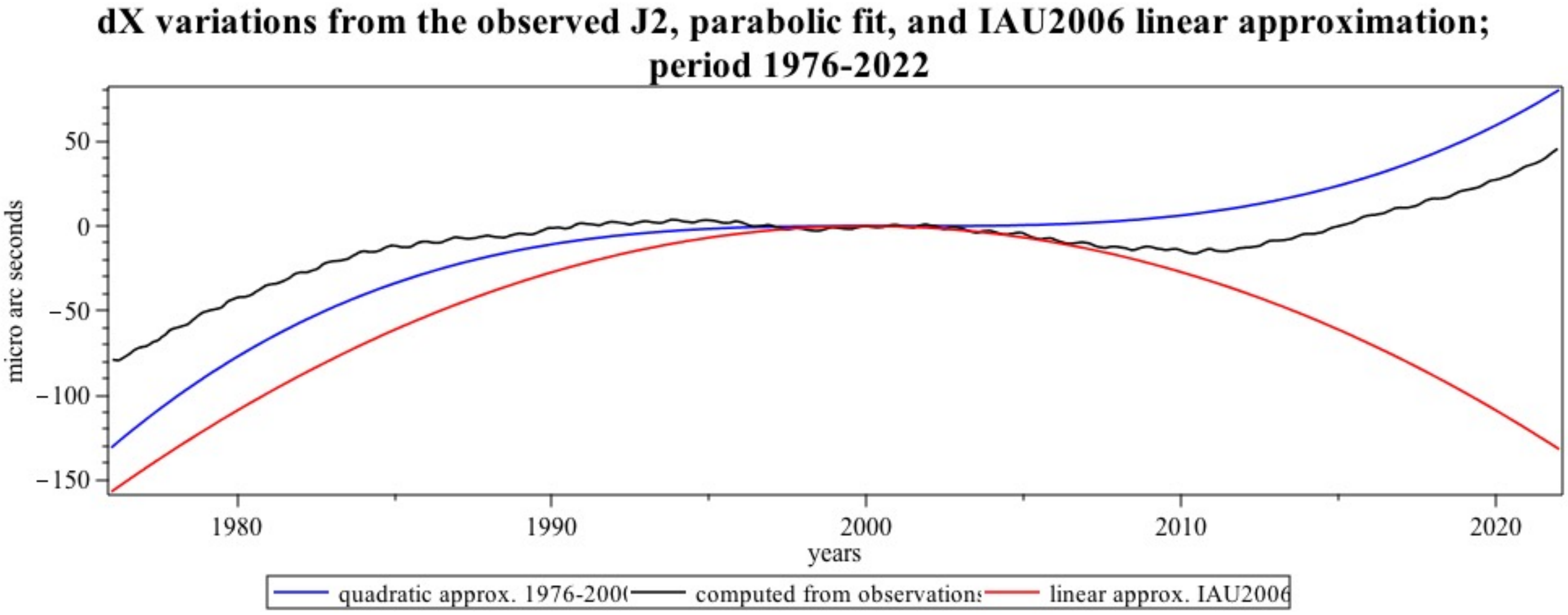
Precession 3: effects of the assumptions on the H variation

- **The linear H trend assumed in IAU2006 contributes a quadratic term to the precession polynomial in longitude (implicit in its final expression) whereas a quadratic fit of H contributes a cubic parabola to it, as shown in the figure together with the difference of both – *increasing and > 50 µas since about 2010***



Precession 3: effects of the assumptions on the H variation

- **dX variations computed numerically from the observed J_2 change are closer to the former cubic approximation than to the parabolic one –** ***respective differences are tens and hundreds µas***



Prospects of improvement the precession models by updating the H variation function

The impact on global WRMS is lower that that of the linear trend

Some potential actions for improving the precession model at short term may be:

1. ***Leave the theory as it is (no action) or update the linear trend of H***
2. ***Replacing the linear trend of H by a quadratic fit*** **– closer to reality**
3. ***Return to an old-style model with constant value of H***

A reflection on updating the model

Pros: model would be closer to actual Earth and more accurate

Cons: H variations depend on long-term observations – though linear variation too

NUTATION: results and prospects of short-term model improvement

- After correcting precession, the only possibility for reducing the unexplained variance of CPO time series would be improving the nutation part of PN models
- No corrections to the IAU2000 nutation theory have been adopted or recommended by the IAU, IAG or IERS so far – *nor used in practice*
- The only IERS recommendation is using the 2007 Lambert's empirical model intended to remove the effect of one of the free oscillations of the Earth axis on nutations, usually named free core nutation (FCN); the model is updated yearly and described in the IERS Conventions (2010)
- FCN models *depend on future observations and require periodic updating because the motion is excited by geophysical sources.*
- There are two main kinds of models according to their derivation:
    1. Most common: Fit a time-varying amplitude to the FCN oscillation by a sliding window approach (SWA) *(e.g., Lambert's, Malkin's or Belda et al's models)*
    2. Fitting constant amplitudes for a chosen set of frequencies in a band around FCN (BA) *(e.g., L Petrov 2007 or Nurul-Huda et al 2020)*

NUTATION: results and prospects of short-term model improvement

Supplementing IAU2000 with FCN models allows a drastic reduction of the WRMS

Example from Heinkelmann et al invited presentation at IAU 2018 GA

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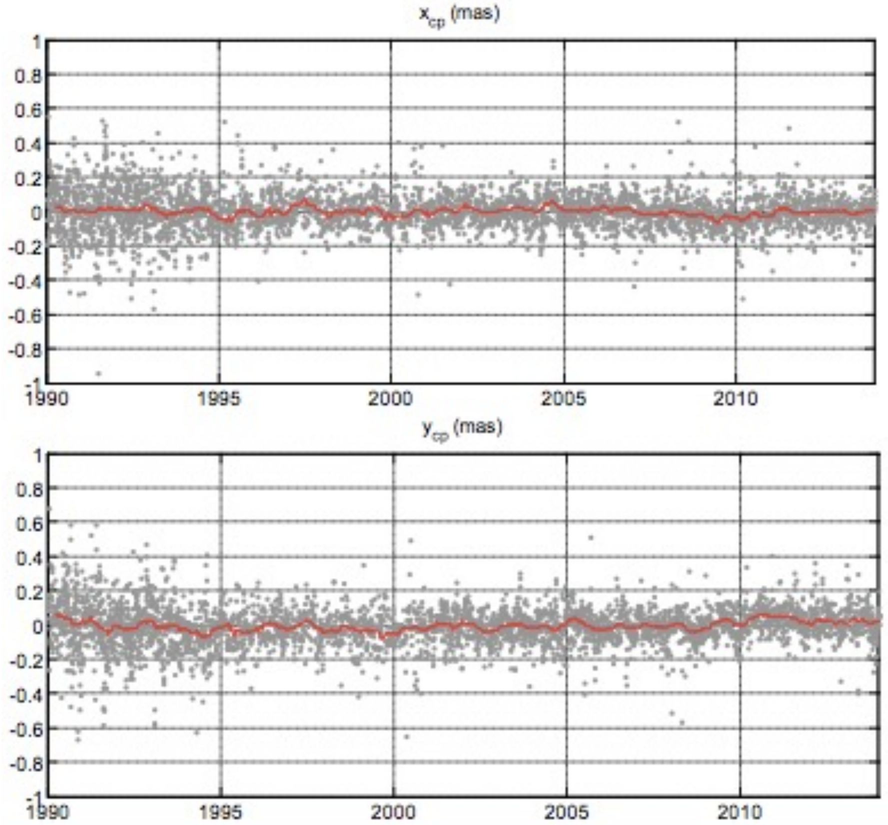
IAU2006/2000A+FCN[1] - EOP finals differences



From 1993.0 on	Shift (µas)	Drift (µas/yr)	Rms (µas)
dX	1.5	- 0.6	97.7

From 1993.0 on	Shift (µas)	Drift (µas/yr)	Rms (µas)
dY	- 21.1	3.0	102.2

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1) Belda et al. (2016), J. Geodyn. 94, pp. 59

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

NUTATION: results and prospects of short-term model improvement

However, using FCN models is not the only possibility for reducing the CPO variance:

- **Improving the accuracy of the forced nutation models is a need recognized by IAU and IAG resolutions; besides, it is feasible to a good extent at short term**
- **Many analyses have shown that the amplitudes of the main nutation components can be improved, reducing the WRMS in a good amount**
- ***Other advantage is that those corrections do not depend on future observations because they are mainly based on astronomical forcing (with known expansions)***
- **Next, we illustrate with some examples how the WRMS can be reduced by using different kinds of corrections to the current IAU2000 nutation model, either alone or jointly.**
- **For the clarity and simplicity sakes, we begin showing tests limited to usn2020 series in 1990-2020 *(Ferrándiz et al EGU2021)***

Test case 1 a: removing a selected FCN model
(Only possibility suggested in the current IERS conventions)

- **Pros:**
  WRMS of time series are greatly reduced.
  Users may choose among several existing good models or derive their own ones by any available method (sliding window or band approaches in the main.
- **Cons:**
  models must be derived from past observations, are affected of larger latency than CPO, and predicting future FCN evolution entails rather high uncertainty ***(see however the prediction method developed by Belda et al 2018)***
- **Performance:**
  Detrending lowers the mean WRMS from 175 to 164 ***μas***
  Subtracting the FCN B16-21 model ***(an extension of the 2016 Belda et al FCN model B16)*** **of residuals after detrending produces mean WRMS about 83** ***μas;***
  the mean drop is about 50% of detrended WRMS, 82 μas,

Test case 1 b: Empirical correction of the amplitudes of given sets of lunisolar (LS) nutation terms

- **Corrections to the amplitudes of various sets of LS nutation terms have been derived in the last years. Results depend on several factors:**
- **Selection of Terms:**

  Most of the corrections limit to a sub-set of the 21 periods used by *Herring et al 2002* for fitting the MHB2000 theory, although more periods may be used.

  ***Belda et al 2017* started with 179 LS periods, reduced to 14 eventually using strict criteria on errors**

  In the next experiment we show the performance of a widder set of 28 periods that allow separation and do not seem to exhibit overfit
- **Fit approach: Amplitude fits can be either unconstrained *(Malkin 2016, Belda et al 2017)* or constrained (re-fitting of the basic Earth parameters of IAU2000, either indirect using previously derived session-wise solutions or direct, working with VLBI time delay data)**
- **Performance of the set of 28 LS periods:**

  the mean drop of the WRMS of detrended series achieves 22 *µas* *(164 -> 142)*

Test case 1 b2: Combination of LS + FCN corrections

- **The correction procedure in 1b may be improved by combining it with 1a. A new, ad hoc FCN model can be derived for the residuals after applying the LS corrections, and then removed from residuals**
- **Performance:**
  The final WRMS after successive application of the two corrections to detrended usn2020 is 82 *μas*
  Notice that this value is similar to that of case 1a (FCN removal only), but has the advantage that a portion of the gain depend on the improvement of forced nutations

Test case 1 c: correction of amplitudes of planetary (PL) nutation terms

- An **analytical solution for the Oppolzer terms of planetary origin** was derived by *Ferrándiz et al 2018* and showed that some non-negligeable planetary contributions are not included in IAU2000
- Theoretical values of some amplitudes may be refined by fitting to CPO series, whereas the improvement of others requires relying more on theory when their separation by Fourier analysis is problematic.

  In any case, attention must be paid to avoid misfit and overfit

Performance:

- Comparable to that of LS correction, with a **mean WRMS drop of about 25 μas**

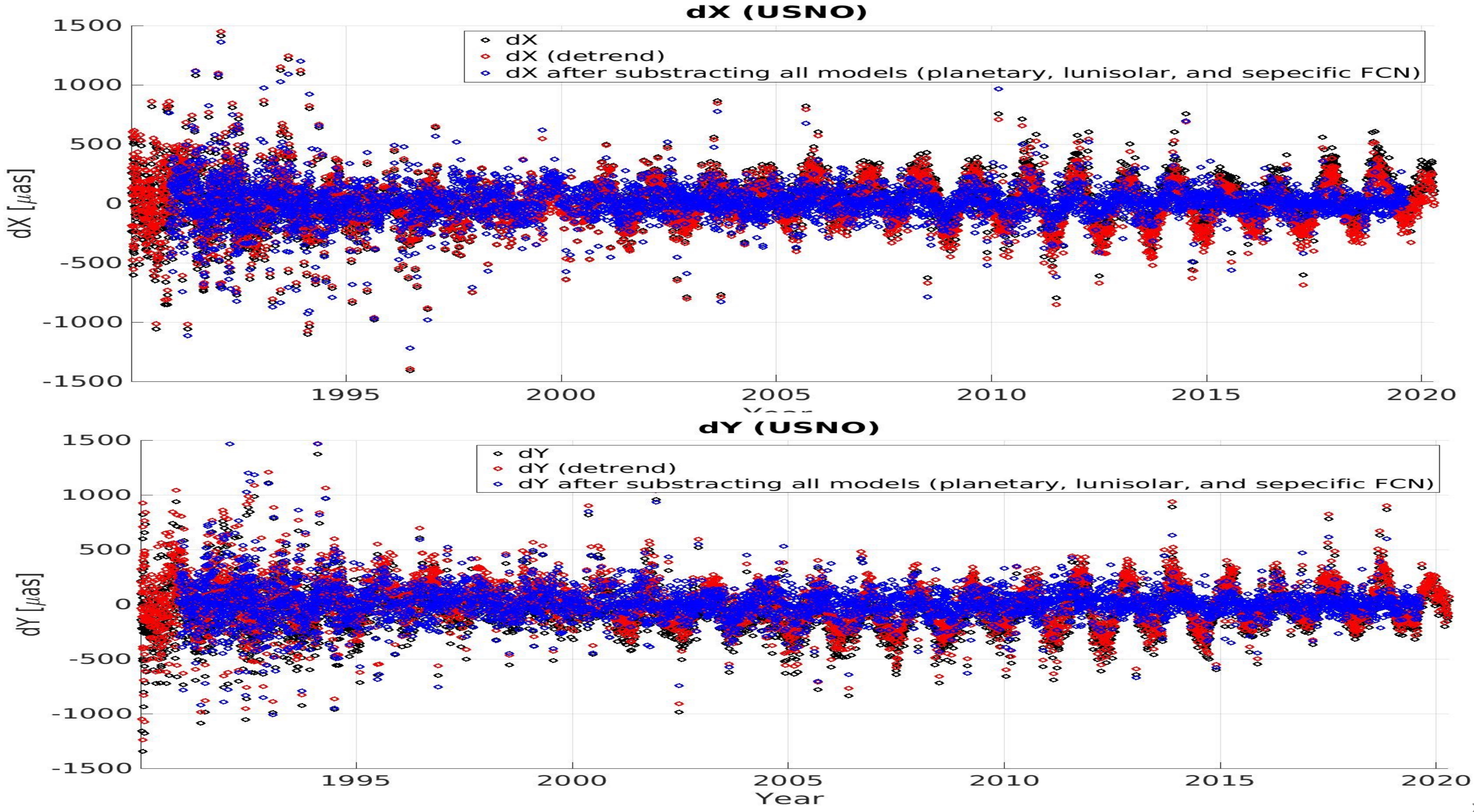
Test case 1 d: Combination of LS + PL + FCN corrections

- Similar to 1 b2, i.e., LS+PL corrections supplemented with FCN

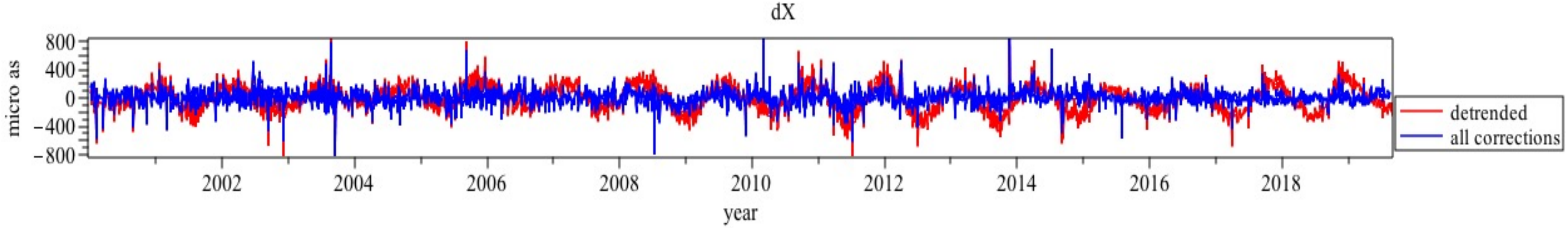
Performance:

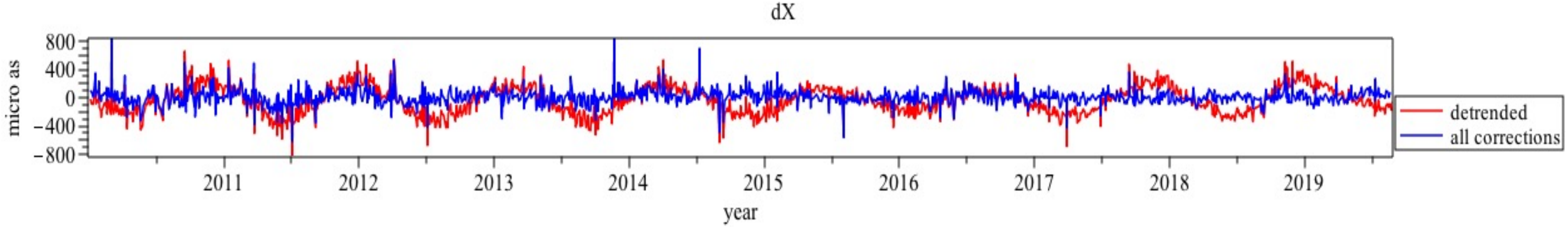
- the **final mean WRMS gets down to 80 μas**

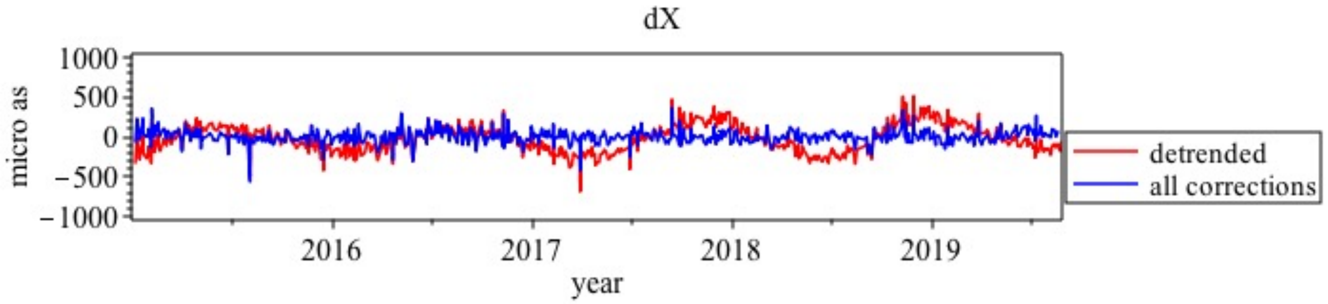
Plots of the CPO input data and residuals after corrections, test case 1 d



Details of detrended and corrected dX series in shorter time intervals







(vertical display range is restricted)

Summary of all test 1 cases

Reduction of the unexplained variance of CPO time series: approaches and mean WRMS drop		
Test source data: usn2020a session-wise solution, Jan. 1990 – Dec. 2020. WRMS (dX, dY) = (173, 177) µas		Initial mean WRMS : **175 µas**
DETRENDING (linear regression). **WRMS of residuals mean drop: 10 µas (dX, dY) = (161, 167) µas**		Mean WRMS of res.: **164 µas**
CURRENT CONVENTIONAL PROC. A. Applying **NO CORRECTION** to IAU2000/2006	***COMMON RESEARCH APPROACH*** A. **EMPIRICAL CORRECTIONS to NUTATION AMPLITUDES of *LUNISOLAR BLOCK (LS)***	***FULLY NEW APPROACH*** A. **ANALYTICAL CORRECTIONS to NUTATIONS AMPLITUDES of *PLANETARY BLOCK (PL)***
Mean WRMS of residuals: 164 µas	Drop with **new 28 periods** set: **22 µas** **Mean WRMS of residuals: 142 µas**	Mean drop with **15 periods**: **25 µas** (dX,dY)=(20,30) µas
B. **REMOVING FCN MODEL** derived from **detrended CPO**	B. **REMOVING FCN MODEL** derived from residual after **LS correction**	B. **EMPIRICAL CORRECTIONS** to NUTATIONS AMPLITUDES of **LUNISOLAR BLOCK** (LS)
Drop of **82 µas** with the **new B16-21 model**	Additional drop of **60 µas**	**ACCUMULATED** WRMS mean drop : **45 µas** **Mean WRMS of residuals: 119 µas**
		C. **REMOVING FCN MODEL** derived from residual after **PL+LS correction**
		Additional drop of **40 µas**
FINAL MEAN **WRMS of residuals: 82 µas**	FINAL MEAN **WRMS of residuals: 82 µas**	FINAL MEAN **WRMS of residuals: 80 µas**

Nutation: results of re-fitting amplitudes of selected forced nutation periods

In this example we show the WRMS results for a subset of the various VLBI solutions used for precession formerly, after fitting a selected set of lunisolar and planetary components. *Time interval is the same, 1984-2021*

CPO	Solution	No. Obs.	WRMS raw input data	WRMS after detrending	WRMS with further lunisolar corrections	WRMS with further planetary corrections	Accumulated WRMS gain %
dX	bkg2020a	5989	214,6	172,6	163,1	153,0	29
dX	gsf2020a	6431	196,0	166,1	156,0	145,4	26
dX	usn2021c	5613	183,4	160,8	150,6	141,7	23
dX	ivs19q4X	4215	173,1	152,9	128,4	116,0	33
dX	ivs20q2X	4266	205,0	168,0	132,8	119,1	42
dY	bkg2020a	5989	176,1	175,6	163,5	153,9	13
dY	gsf2020a	6431	184,5	172,8	158,7	149,5	19
dY	usn2021c	5613	176,6	165,2	151,8	143,4	19
dY	ivs19q4X	4215	174,0	159,3	135,0	122,7	29
dY	ivs20q2X	4266	158,2	145,9	130,3	119,3	39

- **The largest WRMS decrease happens in IVS combined solutions**

Summary of CPO corrections assessed here

We present results for the former input data that include:

- **First step: detrending *(always done)***
- **Next steps: Correcting nutations with three different procedures:**
  1. **Fitting only a reduced set of 15 periods of lunisolar origin (forced nutation model Fn 1)**
  2. **Fitting a wider set of 21 periods of various origins (forced nutation model Fn 2)**
  3. **Using forced model Fn 1 jointly with an FCN model (Fn 1 + FCN)**
- **Results are summarized in a table displaying both CPO and WRMS for uncorrected and all corrected cases.**
  Considering the mean of VLBI solutions scatters, the WRMS of raw series may be about halved by correcting amplitudes of 15 periods and using an FCN model (results in upper table of next slide are for a "band" FCN model)

Correcting nutation: WRMS for various data and models, period 1984-2021

Input data		dX WRMS					Input data	dY WRMS				
	No obs	Raw	Detrend	Det + fn1	Det + fn2	Det + fn1+FCN		Raw	Detrend	Det + fn1	Det + fn2	Det+ fn1+FCN
usn2021c	5613	183.4	160.8	150.7	141.8	91.5	usn2021c	176.6	165.2	151.9	143.4	93.2
gsf2020a	6431	196.0	166.1	156.0	145.5	94.8	gsf2020a	184.5	172.8	158.7	149.5	96.7
bkg2020a	5989	214.6	172.6	163.1	153.0	105.5	bkg2020a	176.1	175.6	163.6	154.0	105.9
opa2021a	7027	211.1	205.8	196.0	190.0	155.2	opa2021a	223.1	201.1	192.9	185.4	150.3
gsfc20q2	4229	176.9	167.6	157.8	149.8	100.1	gsfc20q2	190.8	177.8	161.0	152.8	109.3
bkg20q2	4209	177.8	167.3	159.8	149.8	100.3	bkg20q2	186.2	174.9	160.3	150.8	104.0
dgfi20q2	4034	172.6	162.4	154.4	144.6	96.5	dgfi20q2	179.0	168.8	154.0	144.6	96.0
ivs20q2X	4266	205.0	168.0	133.1	119.5	86.4	ivs20q2X	158.2	145.9	130.4	119.4	73.2
ivs19q4X	4215	173.1	152.9	129.1	116.4	84.0	ivs19q4X	174.0	159.3	135.0	122.7	84.4
IERS14C04	13879	180.2	164.8	157.8	153.9	115.0	IERS14C04	173.2	163.2	154.6	151.9	114.9
finals	10910	150.1	147.0	141.3	131.8	75.9	finals	156.3	138.3	125.2	92.9	75.5
JPL2	8729	174.6	147.7	139.1	131.2	54.2	JPL2	172.3	154.0	140.1	134.4	54.9
From 8 VLBI solutions:							*From 8 VLBI solutions:*					
MEAN		187.4	164.7	150.5	140.1	94.9	MEAN	178.2	167.5	151.9	142.2	95.3
Std Dev		*15.9*	*6.0*	*12.6*	*14.1*	*7.3*	*Std Dev*	*9.9*	*10.7*	*12.5*	*13.6*	*11.9*

COMPARISON WITH Fits20 solution as reported in Zhu et al (2021), period 1984-2020.
Our forced nutation models 1 & 2 behave better for ivs19 and C04, but used BA FCN looks worse than SWA below

ivs19q4X	4216	173.2		154.1		68.9	ivs19q4X	173.9		160.7		71.0
IERS14C04	13465	171.7		161.0		108.8	IERS14C04	175.1		163.5		106.1

Correcting nutation: WRMS for various data & models, 1984-2021 (FCN : upper: Band,Lower: SW)

Input data		dX WRMS					Input data	dY WRMS				
	No obs	Raw	Detrend	Det + fn1	Det + fn2	Det + fn1+FCN		Raw	Detrend	Det + fn1	Det + fn2	Det+ fn1+FCN
usn2021c	5613	183.4	160.8	150.7	141.8	91.5	usn2021c	176.6	165.2	151.9	143.4	93.2
gsf2020a	6431	196.0	166.1	156.0	145.5	94.8	gsf2020a	184.5	172.8	158.7	149.5	96.7
bkg2020a	5989	214.6	172.6	163.1	153.0	105.5	bkg2020a	176.1	175.6	163.6	154.0	105.9
opa2021a	7027	211.1	205.8	196.0	190.0	155.2	opa2021a	223.1	201.1	192.9	185.4	150.3
gsfc20q2	4229	176.9	167.6	157.8	149.8	100.1	gsfc20q2	190.8	177.8	161.0	152.8	109.3
bkg20q2	4209	177.8	167.3	159.8	149.8	100.3	bkg20q2	186.2	174.9	160.3	150.8	104.0
dgfi20q2	4034	172.6	162.4	154.4	144.6	96.5	dgfi20q2	179.0	168.8	154.0	144.6	96.0
ivs20q2X	4266	205.0	168.0	133.1	119.5	86.4	ivs20q2X	158.2	145.9	130.4	119.4	73.2
ivs19q4X	4215	173.1	152.9	129.1	116.4	84.0	ivs19q4X	174.0	159.3	135.0	122.7	84.4
						Band / SW						Band / SW
IERS14C04	13879	180.2	164.8	157.8	153.9	115. / 108.	IERS14C04	173.2	163.2	154.6	151.9	115. / 106
finals	10910	150.1	147.0	141.3	131.8	75.9	finals	156.3	138.3	125.2	92.9	75.5
JPL2	8729	174.6	147.7	139.1	131.2	54.2	JPL2	172.3	154.0	140.1	134.4	54.9
From 8 VLBI sol							*From 8 VLBI sol*					
MEAN		187.4	164.7	150.5	140.1	94.9	MEAN	178.2	167.5	151.9	142.2	95.3
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Conclusions and outlook

- The use of certain sets of semi-empirical corrections to the precession and forced nutation models IAU2006 and IAU2000 may bring the WRMS of each CPO from over 170 *μas* to about 120 *μas*, for combined IVS solutions,
- Agreeing and implementing a suitable set should not take long
- Using supplemental FCN models the WRMS lowers to about 84 to 70 *μas;* *FCN models obtained by sliding window approach performed better than band approach ones in our tests*
- *Our tests bolster than a more complete physical background of models helps to explain more variance, either for forced or free nutations*

Take away message:

- ***WRMS may be halved by correcting CPO this way***